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POPULATION MOBILITY AND EFFECIENCY IN THE PROVISION OF REGIONAL PUBLIC GOODS

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I. Introduction

The purpose of the present paper is to extend the analysis of public goods to situations where population is variable. The focus of the analysis will be a system of local governments, or a federation, where population movement is important and where boundary spillovers of benefits from local public goods may be generated. The paper will develop a simple model of local public good provision and this model will be used to derive efficiency conditions for population movement between regions and for optimal production of public goods. A behavioural model of free migration will be presented to determine whether free migration of people between fiscal regions will lead to efficiency in locational choice. The general result that will be established is that the free market does not lead to an optimal allocation of population among fiscal regions.

In recent years there has been a renewed interest in the question of optimal migration between different fiscal regions. Prior to Samuelson's integration of public goods into Walrasian general equilibrium theory[1] there was a heated exchange in the literature on the relation between equalization grants to regional governments within a federation, effected by the central government, and population movement. This was an exchange between Anthony Scott and James Buchanan[2] which took place in the early fifties and the argument focused upon the question: does "the market" lead to efficient movement of people as between regions? Buchanan concentrated upon the problems associated with a non-benefit system of taxation and he argued that equalization payments, or some arrangements equivalent thereto, would be required both on efficiency and equity grounds. The substance of the Scott-Buchanan exchange will not be reviewed here. It will be noted however that neither of the participants set out a formal model to deal with the problem and that both of them seemed not to be aware of the importance of public goods to the question of migration. It was a few years later when Samuelson developed the formal efficiency conditions for a model with both private and public goods and with fixed population. These conditions will not be restated here nor will space be taken to explore the characteristics of public goods or to repeat the standard semantics associated therewith.

In 1956 C.M. Tiebout published an important contribution to the problem of preference revelation associated with public goods. Samuelson placed considerable stress upon the market failure problem created by public goods. Tiebout suggested that a system of regional governments, producing differentiated public goods, could provide the means for preference revelation through migration. People could simply move to the region which provides a preferred basket of public goods. In the Tiebout model private incomes are all paid in the form of dividends. Hence, since private goods are assumed to be equally available in all regions the regional public sectors provide the effective basis for choice in migration.

Henry J. Aaron took up the migration problem in a paper published in 1969[3] dealing with the use of a "loss leader" by a community attempting to reduce the per capita cost of public goods. Aaron shows that if a community will gain through the attraction of new residents by the reduction of per capita costs for a given quantity of public goods then its public good provision will reflect this phenomenon. In particular if some specific public good has an important influence on immigration then the community may choose to "oversupply" this public good in order to attract new residents.

In his paper Aaron draws attention to the well-known proposition that a public good will be undersupplied when benefits external to the community and accruing to people beyond its jurisdiction are ignored. He suggests that the "loss leader" or "migration effect" may lead to an expansion of output of public goods which is sufficient to compensate for undersupply due to spillover. The main problem in his treatment of migration effects is that Aaron has no model from which to derive the optimal population for the community and it is only on the basis of some concept of optimal population that a migration policy for the community can be devised.

Another paper which deals with the relationship between population size and public goods is an article by James Litvack and Wallace Oates (Litvack and Oates, 1970). The purpose of their paper is to relate the demand for public goods to the per capita tax price, which depends inversely upon the size of population. They develop a model focusing upon the demand for public goods and they use it to analyze the degree of fiscal centralization in the United States. They do not deal with the optimal population problem as such but rather discuss the implications of different population size for public goods demand.

In 1965 James Buchanan published a seminal contribution to the economic theory of clubs (Buchanan, 1965). This paper shows how a group of individuals could improve their welfare position by forming a club to finance the provision of a collective club good. Buchanan demonstrates that the efficient club size and collective good output are simultaneously determined. In two recent papers co-authored by Buchanan (Buchanan and Wagner, 1970 and Buchanan and Goetz 1972) the problems of migration and public goods are taken up again. The following remarks will relate to the later of these two papers, published by Buchanan and Goetz. The insights that Buchanan developed in the context of his theory of clubs appear afresh here, in particular those concerning the simultaneity of optimal public good provision and optimal migration.

Buchanan and Goetz (B-G) provide a re-examination of the Tiebout model and attempt to determine whether utility-maximizing individuals will make decisions concerning where they will live in a manner consistent with overall efficiency. The basic point that they make is that the free migration associated with the Tiebout adjustment process is unlikely to lead to an efficient allocation of population among fiscal regions. But in making this important point they have not explicitly presented a model of welfare maximization from which to draw their inferences.

The B-G paper deals with efficient migration in a system of regional government. The problem of boundary spillover is ignored but explicit attention is given to congestion of public goods and to the spatial dimension, by which is meant the contribution of locationally fixed factors of production, and the ownership of such fixed factors. Since migration and its efficiency properties are at the heart of the discussion it is not surprising to find that the spatial dimension is important. B-G argue that there are external effects associated with the individual decision to relocate and that these external effects appear on both the benefit and the taxation side of public good provision. They point out that these external effects are ignored by the decision-maker, who is responsive only to differences between regions in factor payments and in consumer surplus from public good provision.

In the present paper the optimality of "voting-by-foot" is re-examined in the context of a set of formal models. The analysis below confirms the B-G finding that Tiebout-type migration is not led by an "invisible hand" to achieve optimality. In Section II a model is developed to examine public goods and variable population for a single region. The model is adapted to deal with a world of identical individuals and with a world where tastes are differentiated. The section provides a framework of analysis for the rest of the paper.

In Section III the analysis is extended to a two-region world with identical tastes. Efficiency conditions are developed for public and private goods and for optimal population size and it is shown that free migration will not replicate these conditions. Federal subsidies are then examined as potential instruments of federal optimization. Section IV allows for differences in tastes between individuals, again within the context of a two-region world. The focus in Section IV is upon the use of the public-private good "product mix" by regional governments to attain a desired population size. Once again non-optimal population movements are created and there is scope for corrective federal subsidies. Section V concludes the paper.

II. Optimality in Population and Public Goods for a Single Region

In this section attention is focused upon a single region in which two goods, a regional public good and a private good, are provided. Private goods are provided by competitive private industry while the public good, assumed for the present to be a pure public good, is provided by the regional government. Let it be assumed initially that all residents of our single region are identical and that there are many non-residents who would like to enter the region. In our initial model[4] there is regulated in-migration and the problem is to determine the optimal population size for the region.

The basic model is now given by the following equations:

$$(1) \qquad U = U(\tfrac{X}{N}, R) \ ,$$

$$(2) \qquad Y = X + R \quad , \text{ and}$$

$$(3) \qquad Y = \hat{Y}(N)$$

Equation (1) is the social welfare function for the region and it is defined in terms of per capita consumption of public and private goods. Since there are no imports or exports allowed in this model all production will be consumed in the region.[5] Production of private and public goods will be denoted by X and R, respectively. It is assumed that there are two factors of production, land and labour, and that property rights in the region's scarce endowment of land are shared equally among all residents. Private and public goods are technically the same good,[6] for simplicity, so that $Y(=X+R)$ measures total output in equation (2). Equation (3) is the production function which relates regional output to the size of the resident population.

In order to find the optimal values for X, R and N we may set up the following Lagrangian expression:

$$(4) \quad L = U(\tfrac{X}{N}, R) + \lambda_1[Y-X-R] + \lambda_2[\bar{Y}(N)-Y]$$

This expression yields the following first order conditions for optimality:

$$(5) \quad \frac{\partial L}{\partial X} = \frac{U_1}{N} - \lambda_1 = 0 \ ,$$

$$(6) \quad \frac{\partial L}{\partial R} = U_2 - \lambda_1 = 0 \ ,$$

$$(7) \quad \frac{\partial L}{\partial Y} = \lambda_1 - \lambda_2 = 0 \ , \quad \text{and}$$

$$(8) \quad \frac{\partial L}{\partial N} = \frac{-X}{N^2}U_1 + \lambda_2\frac{\partial \bar{Y}}{\partial N} = 0 \ .$$

From equations (5) and (6) we get the Samuelson condition $N\frac{U_2}{U_1} = 1$ and from (5), (7) and (8) we get

$$(9) \quad \frac{X}{N} - \frac{\partial \bar{Y}}{\partial N} = 0 \ ,$$

which states that the average consumption of private goods must be equal to the marginal product of labour.[7] This equation is the first order condition for optimal population. If the second order conditions for welfare maximization are satisfied then the complete set of first order conditions enables us to solve for optimal population size.

If it is assumed that all residents and potential residents are identical then it is easy to interpret the social welfare function in (1) as the utility function of the representative resident. Here and in much of the rest of this paper the social welfare function will be defined in terms of the welfare of the representative person. Social welfare will be defined to depend upon per capita income rather than total income. If A and B are two fiscal regions for which total cardinal utility is the same then we shall define the "happier" region as the one which has the higher per capita level of utility. An alternative would be to actually specify an objective function in terms of total utility If the objective function in (4) were altered to NU the (unsurprising) effect would be to substitute the inequality

(10) $\frac{X}{N} - \frac{\partial \widetilde{Y}}{\partial N} > 0$

for the equality given in (9).[8]

Imperfect Migration

Up to this point the paper has dealt with a set of identical individuals comprising residents and potential residents and potential residents of our focal region. The objective was to maximize regional welfare in a situation where population size could be directly controlled through migration policy. It will now be assumed that migration to and from the region is unrestricted but that potential residents are not identical. The region that is of interest is small in relationship to the rest of the fiscal system and this enables us to assume that the relevant economic variables in other regions remain invariant to changes that are introduced in the following analysis. One additional relationship is added to the model to be presented here and that is the "mobility function" in (11):

$$(11) \quad N = \widetilde{N}(W - \frac{R}{N}, R) \ ,$$

where N is the population of the region and W is per capita income. The term $\frac{R}{N}$ refers to the required tax payment per person so that $W - \frac{R}{N}$ is income in private goods net of tax payments. Equation (11) may be rewritten as (12)

$$(12) \quad N = \widetilde{N}(\frac{X}{N}, R).$$

The problem is now to maximize the following Lagrangian expression:

$$(13) \quad L2 = U(\frac{X}{N},R) + \lambda_1(Y-X-R) + \lambda_2[\widetilde{Y}(N)-Y] + \lambda_3[\widetilde{N}(\frac{X}{N},R)-N].$$

The first order conditions, associated with X, R, Y and N, are as follows:[9]

$$(14)\quad \frac{\partial L_2}{\partial X} = \frac{U_1}{N} - \lambda_1 + \lambda_3\frac{\widetilde{N}_1}{N} = 0 \ ,$$

$$(15)\quad \frac{\partial L_2}{\partial R} = U_2 - \lambda_1 + \lambda_3\widetilde{N}_2 = 0 \ ,$$

$$(16)\quad \frac{\partial L_2}{\partial Y} = \lambda_1 - \lambda_2 = 0, \text{ and}$$

$$(17)\quad \frac{\partial L_2}{\partial N} = -\frac{X}{N^2}U_1 + \lambda_2\frac{\partial \widetilde{Y}}{\partial N} - \lambda_3\left[\frac{X}{N^2}\widetilde{N}_1 + 1\right] = 0.$$

If we solve for λ_1, λ_2 and λ_3 then we can write:

$$(18)\quad \left(\frac{X}{N} - \frac{\partial \widetilde{Y}}{\partial N}\right)\frac{dN}{dR} = SN-1 \ ,$$

where $S(=\frac{U_2}{U_1})$ is the marginal rate of substitution between private and public goods and $\frac{dN}{dR}$ is the total derivative of N with respect to R, indicating how the resident population responds to a change in the output level of the public good.

The region cannot directly control its population size and hence it is unlikely that the condition $\frac{X}{N} = \frac{\partial \widetilde{Y}}{\partial N}$ will be satisfied. Either the region will be overpopulated $(\frac{X}{N} - \frac{\partial \widetilde{Y}}{\partial N} > 0)$ or it will be underpopulated $(\frac{X}{N} - \frac{\partial \widetilde{Y}}{\partial N} < 0)$. If public good provision has no effect on population size (i.e. $\frac{dN}{dR} = 0$), so that migration flows are unrelated to the level of public good output, then the Samuelson condition (i.e. $SN = 1$) should be satisfied. That is to say the summed marginal rates of substitution should be equated to the cost of public goods (i.e., unity, in our model). On the other hand if an increase in public good output attracts new residents (i.e., $\frac{dN}{dR} > 0$) and if the region is overpopulated (i.e., $\frac{X}{N} - \frac{\partial \widetilde{Y}}{\partial N} > 0$) then the public good should be "underproduced" so that $NS > 1$. If the region were underpopulated (and if $\frac{dN}{dR} > 0$) then the public good should be "overproduced" (i.e. $NS < 1$). The analysis here suggests that free migration may lead to optimal violation of the Samuelson condition if the social welfare function is to be maximized. In the previous model, with controlled migration, the level of population is directly controlled and so an optimal level of population canbe achieved.

The tax-pricing system implicit in our analysis is very simple. If our optimality conditions require that the Samuelson condition be satisfied (i.e., $NS = 1$) then the tax per person is $\frac{Y-X}{N} = \frac{R}{N} = RS = T$. This means that the tax price per unit of R is the same on both marginal and intramarginal units and that the unit price is equal to the (assumed) uniform marginal rate of substitution. But if our optimality conditions require that $NS-k=1$, where $k \gtrless 0$, then the tax per person is $\frac{Y-X}{N} = \frac{R}{N} = \frac{RS}{(1+k)} = T$. If equation (18) is rewritten as

(19) $$k = NS - 1$$

then if $k > 0$ it may be regarded as a measure of induced overpopulation resulting from a small change in public good output. Note that the public good is pure, or completely non-rivalrous, so that there is no congestion in the sense of dissipation of benefits from the public good. Congestion appears only in the form of Ricardian diminishing returns.

The present model focuses upon free migration in a world where people have different tastes. Migration has been introduced through our demand for residence function (i.e., $N = \tilde{N}(\frac{X}{N}, R)$). In spite of our assumption of heterogeneity of tastes the model requires each person in the region to consume identical quantities of the public and private good. A brief discussion and interpretation of the Lagrangian multipliers will follow and an alternative optimization scheme with differential consumption will be sketched.

Equations (14) to (17) give four of the seven first-order conditions derived from the Lagrangian expression given in (13). The three remaining conditions are those associated with the Lagrangian multipliers λ_1, λ_2 and λ_3. Since (16) gives $\lambda_1 = \lambda_2$ attention will be concentrated on λ_1 and λ_3. It is well known that "the Lagrange multipliers at the solution measure the sensitivity of the optimal value of the objective function $F^* = F(X^*)$ to variations in the constraint constants" (Intriligator, 1971, p. 36). This means that we may interpret λ_1 as the contribution to social welfare of a small increase in private or public good output. If we solve for λ_1 we get

$$(20) \quad \lambda_1 = \frac{U_1}{N}(NS - \frac{\tilde{N}_2}{\tilde{N}_1}N)/(1 - \frac{\tilde{N}_2}{\tilde{N}_1}N),$$

where $S = \frac{U_2}{U_1}$. In the special case where $k = NS-1 = 0$ then $\lambda_1 = \frac{U_1}{N}$, which is the increase in social welfare resulting from the availability of an extra unit of private goods. As such, λ_1 may be interpreted as the marginal social utility of private goods. Furthermore, when $k = 0$ then $\lambda_1 = \frac{U_1}{N} = U_2$ which follows from the Samuelson condition. When $k \neq 0$ then the marginal utility of an additional unit of output will depend to some extent upon the induced change in population.

The Lagrange multiplier λ_3 in (13) is the contribution to social welfare resulting when population is increased by one person. In the present model the contribution comes in the form of both increased output and consumption of private goods. At the constrained welfare maximum the value of λ_3 is given by

$$(21) \quad \lambda_3 = \frac{U_1}{N}(NS-1)/(\frac{\tilde{N}_1}{N} - \tilde{N}_2) = \frac{U_1}{N}k/(\frac{\tilde{N}_1}{N} - \tilde{N}_2).$$

If $k = 0$ then $\lambda_3 = 0$ so that a small change in the size of the resident population has a negligible effect on social welfare.[10] The denominator in (21) is a measure of the responsiveness of migration flows to a switch from private to public goods. If $h = (\frac{\tilde{N}_1}{N} - \tilde{N}_2) < 0$ then a shift of one unit of output from private to public goods will attract more people. If h and k are both positive then λ_3 will be positive so that additional people will increase the level of social welfare. If h and k have opposite signs then λ_3 will be negative and in-migration will lower social welfare. Each of the Lagrange multipliers is measured in utility terms so that λ_3 reflects the change in social welfare associated with an additional person while λ_1 is a measure of the marginal social utility of a unit of output in the form of private goods. Therefore λ_1 serves to convert additional units of private goods into units of utility so that for any increase in supply of private goods, ΔX, we can write $\Delta U = \lambda_1 \Delta X$. This means that the reciprocal of λ_1 may be used to convert from units of utility into units of private goods. In particular, it means that we can write $\frac{\lambda_3}{\lambda_1}$ as the marginal social value of an additional person, measured in terms of private goods. If the region is overpopulated then λ_3 will be negative and $\frac{\lambda_3}{\lambda_1}$ may be interpreted as the tax that must be levied against potential in-migrants to avoid a loss of welfare.

This analysis suggests that a discriminatory tax policy for potential in-migrants could in theory be used as an additional policy instrument. An underpopulated region would offer a subsidy to potential residents while an over-populated region would impose a tax upon new residents. Such discriminatory taxation of new residents is basically inconsistent with the concept of free migration with which we are concerned. Also, it is difficult to handle discriminatory taxation in the framework of a model based upon the assumption of equal consumption of private and public goods. For these reasons little else will be said with respect to "border pricing", or discriminatory treatment of migrants in what follows. Population size will be assumed to be subject to indirect control through variation in the private-public good mix.[11]

III. Inter-regional Migration and Pure Public Goods in a Two-Region World.

In this section a two-region model is introduced and each region is assumed to produce both public and private goods. As in the previous section private goods are provided by competitive industry while regional governments provide pure public goods. Note that we rule out spillover effects as well as congestion and defer treatment of these problems. Our basic concern in this section is to develop optimality conditions for the two-region model and to determine whether free migration will lead to the fulfillment of these conditions. As before it will initially be assumed that all individuals are identical. In our first model there is a fixed number of identical individuals in the two regions and each person is free to move from one region to the other.

Each region has a fixed amount of land and fixed jurisdictional boundaries. While there is migration between the two regions there is no trade, since there is only one private good produced and consumed in each region. In order to introduce trade it would be necessary to set up a model with two different private goods and we have chosen to make the model as simple as possible by requiring that both regions produce a single private good. Production conditions are simplified by the assumption that the transformation curve between private and public goods is linear in each region.

The utility function for the representative resident of region 1 is given by

$$(22)\quad U = U(\frac{X}{N}, R_1) \ ,$$

where X measures the output of private goods, R_1 the output of pure public goods and N the number of residents. Each of X, R_1 and N is a variable endogenous to our present model. The regional production function may be written

$$(23)\quad Y_1 = \widetilde{Y}_1(N) = X + R_1 \ .$$

It is assumed here that all residents are workers so that N is a measure of the size of the labour force and of the resident population. Each resident is vested with equal property rights in land and this explains why the utility function in (22) has as its arguments the average consumption of private and public goods.

For the representative resident of region 2 we write his utility function as

$$(24)\quad W = W(\frac{Z}{M}, R_2) \ ,$$

where Z measures the output of private goods, R_2 the production of public goods and M the size of population. The regional production function may be written

$$(25)\quad Y_2 = \widetilde{Y}_2(M) = Z + R_2 \ .$$

The total population of the two regions is given by

$$(26)\quad N_0 = N + M.$$

In order to derive efficiency conditions we set up the following Lagrangian expression:

$$(27)\quad L_3 = U(\frac{X}{N}, R_1) + \lambda_1[W(\frac{Z}{M}, R_2) - W^0] + \lambda_2[\widetilde{Y}_1(N) - X - R_1]$$

$$+ \lambda_3[\widetilde{Y}_2(M) - Z - R_2] + \lambda_4[N_0 - N - M].$$

The first order conditions are as follows:[12]

$$(28)\quad \frac{\partial L_3}{\partial X} = \frac{U_1}{N} - \lambda_2 = 0 \quad ,$$

$$(29)\quad \frac{\partial L_3}{\partial R_1} = U_2 - \lambda_2 = 0 \quad ,$$

$$(30)\quad \frac{\partial L_3}{\partial Z_1} = \lambda_1 \frac{W_1}{M} - \lambda_3 = 0 \quad ,$$

$$(31)\quad \frac{\partial L_3}{\partial R_2} = \lambda_1 W_2 - \lambda_3 = 0 \quad ,$$

$$(32)\quad \frac{\partial L_3}{\partial N} = -\frac{XU_1}{N^2} + \lambda_2 \frac{\partial \widetilde{Y}_1}{\partial N} - \lambda_4 = 0, \quad \text{and}$$

$$(33)\quad \frac{\partial L_3}{\partial M} = -\lambda_1 \frac{Z}{M^2} W_1 + \lambda_3 \frac{\partial \widetilde{Y}_2}{\partial M} - \lambda_4 = 0.$$

If we solve for λ_1, λ_2, λ_3 and λ_4 then we get

$$(34)\quad \frac{U_1}{N}(\frac{X}{N} - \frac{\partial \widetilde{Y}_1}{\partial N}) = \lambda_1 \frac{W_1}{M}(\frac{Z}{M} - \frac{\partial \widetilde{Y}_2}{\partial M}) \quad \text{or}$$

$$U_2(\frac{X}{N} - \frac{\partial \widetilde{Y}_1}{\partial N}) = \lambda_1 W_2(\frac{Z}{M} - \frac{\partial \widetilde{Y}_2}{\partial M}) \quad .$$

This is the two-region efficiency condition corresponding to (9) in the previous section. Equation (9) tells us that if each of the bracketed terms in (34) is equal to zero then each of our two regions will have achieved an optimal population size. But it is unlikely that the total population of the two regions, N_0, will be consistent with optimal population sizes in the two regions. It will be completely fortuitous if N_0 is consistent with zero-valued bracketed terms in (34). It is more likely that the most efficient feasible solution is one in which the bracketed terms are either positive or negative. Both regions will be over- or under-populated.

Let us examine how these results are affected if private goods can be redistributed by the central government. If $\widehat{X}$ and $\widehat{Z}$ are used to represent the production of private goods in regions 1 and 2, respectively, while X and Z represent the corresponding quantities consumed then we can write

(35) $\widehat{Y}_1 = \widehat{X} + R_1$,

(36) $\widehat{Y}_2 = \widehat{Z} + R_2$, and

(37) $\widehat{X} + \widehat{Z} = X + Z$.

Equation (35) states that the output of region 1 may be produced in the form of private goods ($\widehat{X}$) or public goods (R_1). A similar interpretation applies to (36). Equation (37) states that the total output of private goods in the two regions ($\widehat{X} + \widehat{Z}$) may be allocated between the two regions for consumption. This allocation will be undertaken in such a way as to maximize the value of the objective function in (27). If the constraints given by (35-37) are substituted for the second and third constraints of (27) then the maximization problem yields

$$(38)\quad \frac{U_1}{N} = \lambda_1 \frac{W_1}{M} = U_2 = \lambda_1 W_2.$$

Note that (38) presents the consequences of a policy of inter-regional redistribution although such a policy does not necessarily equalize the welfare levels of residents in the two regions. Let us interpret λ_1 as the weight assigned to the social welfare level of the second region and let us set its value, arbitrarily, at unity. If all individuals are identical then $U_2 = W_2$ implies that $R_1 = R_2$, but $\frac{U_1}{N} = \frac{W_1}{M}$ does not imply that $\frac{X}{N} = \frac{Z}{M}$. Equation (38) informs us that a policy of inter-regional redistribution will equate the marginal social utility of national output between the two regions. But such a policy will not necessarily equate the marginal private utility of national output since $\frac{U_1}{N} = \frac{W_1}{M}$ is consistent with $U_1 \neq W_1$.

If inter-regional redistribution of income is allowed then the condition for optimal migration becomes simply

$$(39)\quad \frac{X}{N} - \frac{\partial \tilde{Y}_1}{\partial N} = \frac{Z}{M} - \frac{\partial \tilde{Y}_2}{\partial M}$$

Note that the optimality condition refers to the average consumption (not production) of private goods. If an additional constraint were placed upon the maximization problem in (27), to require that the level of utility be the same in the two regions (i.e., $U = W$) then the optimality conditions would become very restrictive. In addition to (38) we would have the requirement that $U_1 = \lambda_1 W_1$ so that if $\lambda_1 = 1$ then $\frac{U_1}{N} = \frac{W_1}{M}$ would constrain solution values to those in which the two regions have the same population, (i.e., $N = M$). If $R_1 = R_2$ and $\frac{X}{N} = \frac{Z}{M}$ (which follows from the requirement that $U_1 = W_1$) then clearly $\frac{\partial Y_1}{\partial N} = \frac{\partial Y_2}{\partial M}$. This means that the marginal product of labour must be identical in the two regions. Unless the two regions are physically identical the imposition of (a) inter-regional and (b) inter-personal redistribution constraints does lead to highly restrictive conditions. These conditions may effectively rule out the possibility of solutions that are of economic interest. In what follows considerable attention will be given to the equal-welfare constraint, which requires that individuals be indifferent between fiscal regions.

The present paper gives short shrift to inter-regional trade in private goods. One analytical advantage to be had from the explicit introduction of trade is that it would facilitate the designation of one traded good as the numeraire good. For example, if our model contained two different private goods with each region producing and consuming both then equilibrium would require the equality of marginal private rates of substitution between the two regions (for the second private good in terms of the numeraire).[13] If all private goods were available at the same price in the two regions then choice of private goods may be independent of location.

Choice of location would then depend upon the output of public goods and the tax price associated therewith and upon factor prices. In the present simple model, with only one private good and no explicit trade, choice of location hinges on the average consumption of private and public goods. The availability of public goods depends upon tastes and resources, but it depends as well upon population size since there are economies of scale in consumption associated with public goods.

We now ask whether free migration will lead to an optimal allocation of population between the two regions. In particular, will free migration lead to the satisfaction of the optimal migration condition given in (34)? It will be assumed that the two regional governments provide public goods in sufficient quantity (R_1 and R_2) to satisfy the Samuelson conditions given in (28)-(31). Each individual will be in location choice equilibrium only if he is indifferent as to whether he lives in region 1 or 2. This means that the free market will lead to $U = W$ so that the two points $(\frac{X}{N}, R_1)$ and $(\frac{Z}{M}, R_2)$ will both lie on the same indifference curve.[14] Let $u = u(a,b)$ then for a small change in a and b we may write $du = u_a da + u_b db = 0$. We may approximate the equation of the indifference curve passing through the points $(\frac{X}{N}, R_1)$ and $(\frac{Z}{M}, R_2)$ by writing

$$(40) \quad U_1(\frac{X}{N} - \frac{Z}{M}) + U_2(R_1-R_2) = W_1(\frac{Z}{M} - \frac{X}{N}) + W_2(R_2-R_1) \quad \text{or}$$

$$(U_1+W_1)(\frac{X}{N} - \frac{Z}{M}) + (U_2+W_2)(R_1-R_2) = 0.$$

In (40) the two points are evaluated in terms of the average marginal utilities (i.e. U_1+W_1 and U_2+W_2).[15]

Let us consider a number of possibilities. First, assume that $U_2 = W_2$, which implies that $R_1 = R_2$ and that $\frac{U_1}{N} = \frac{W_1}{M}$. From (40) it is clear that $R_1 = R_2$ is compatible with locational equilibrium only if $\frac{X}{N} = \frac{Z}{M}$, by our assumption of identical individuals. But if $\frac{X}{N} = \frac{Z}{M}$ then $U_1 = W_1$ and this is consistent with $\frac{U_1}{N} = \frac{W_1}{M}$ only when $N = M$. From (34) it is clear that the marginal product of labour must be the same in the two regions. If the two regions are identical then this is precisely what we would expect, even though labour is not paid its marginal product. Clearly, the case where the regions are identical is the least interesting possibility and the regions are identical from our viewpoint when $R_1 = R_2$ and $N = M$.

As a second case let $U_2 > W_2$ (and $\frac{U_1}{N} > \frac{W_1}{M}$). Migratory equilibrium now requires that $\frac{X}{N} > \frac{Z}{M}$ since $U_2 > W_2$ implies that $R_1 < R_2$. Furthermore $\frac{X}{N} > \frac{Z}{M}$ implies that $U_1 < W_1$, and, together with $\frac{U_1}{N} > \frac{W_1}{M}$, this means that $N < M$. If (34) is rewritten as

$$(41) \quad U_2\frac{X}{N} - W_2\frac{Z}{M} = U_2\frac{\partial\hat{Y}_1}{\partial N} - W_2\frac{\partial\hat{Y}_2}{\partial M}$$

we find that the left-hand side is unambiguously positive but it is not clear that the forces of migratory adjustment will lead to the fulfillment of (41), since migration is responsive to average consumption rather than marginal product. Free migration does not in general lead to the satisfaction of condition (41).

Another way to make the same point is by restating (41) in terms of a simple production function. Assume that the production function is Cobb-Douglas, and the same in both regions so that $Y_i = AL_i^aT_i^b$ $(i=1,2)$, where L_i refers to the size of the labour force and T_i to the amount of land in the i-th region. To make the optimization problem non-trivial let $T_1 \neq T_2$. Now (41) may be rewritten

$$(42) \quad U_2\frac{X}{N} - W_2\frac{Z}{M} = aU_2\frac{Y_1}{N} - aW_2\frac{Y_2}{M} .$$

The point to be made here is that workers are not being paid according to their marginal products and so will not respond to the parameters of the production function.

The third case that remains is that where $R_2 > R_1$ and $\frac{X}{N} < \frac{Z}{M}$. The treatment of this case follows that of the previous one. In all three of these it is assumed that the migrant individual receives his income in the form of the regional average consumption of private and public goods. His private good consumption in region 1 is $\frac{Y_1}{N} - \frac{R_1}{N} = \frac{X}{N}$. Let us now find how sensitive our model is to a change in its underlying assumptions. In particular let it be assumed that free migration is responsive to the supply of public goods in each region and to the marginal product of labour less the average tax price. The relevant private and public good variables in region I are $\frac{\partial Y_I}{\partial N} - \frac{R_I}{N}$ and R_I. This means that there is a migrant group which is treated differently from other people. We retain the assumptions that (22) and (24) are the relevant social welfare functions and we continue to accept (34) as the relevant optimality condition for free migration. Analogous to (40) we now get the condition for equilibrium in choice of location as follows:

$$(43) \quad (U_I+W_I)\left(\frac{\partial \hat{Y}_I}{\partial N} - \frac{R_I}{N} - \frac{\partial \hat{Y}_2}{\partial M} + \frac{R_2}{M}\right) + (U_2+W_2)(R_I-R_2) = 0.$$

Again it is easy to see that (43) in no way leads to the satisfaction of the optimality condition in (34).

Finally, to conclude this section and its treatment of a federation of identical individuals, let us reformulate the analysis, with total utility, rather than average utility, as the objective function. First, we shall develop the optimality conditions and secondly these conditions will be used to evaluate the optimality of free migration. Retaining our earlier notation let the relevant Lagrangian expression be as follows:

$$(44) \quad L_4 = NU\left(\frac{X}{N},R_I\right) + MW\left(\frac{Z}{M},R_2\right) + \pi_I[\hat{Y}_I(N)-X-R_I]$$

$$+ \pi_2[\hat{Y}_2(M)-Z-R_2] + \pi_3(N_0-N-M).$$

The first order conditions are

$$(45)\quad \frac{\partial L_4}{\partial X} = U_1 - \pi_1 = 0 \ ,$$

$$(46)\quad \frac{\partial L_4}{\partial R_1} = NU_2 - \pi_1 = 0 \ ,$$

$$(47)\quad \frac{\partial L_4}{\partial Z} = W_1 - \pi_2 = 0 \ ,$$

$$(48)\quad \frac{\partial L_4}{\partial R_2} = MW_2 - \pi_2 = 0 \ ,$$

$$(49)\quad \frac{\partial L_4}{\partial N} = U - \frac{X}{N}U_1 + \pi_1\frac{\partial \widetilde{Y}_1}{\partial N} - \pi_3 = 0 \ , \quad \text{and}$$

$$(50)\quad \frac{\partial L_4}{\partial M} = W - \frac{Z}{M}W_1 + \pi_2\frac{\partial \widetilde{Y}}{\partial M} - \pi_3 = 0 \ .$$

If we solve for π_1, π_2 and π_3 then we get the familiar Samuelson conditions once again. The optimal migration condition now becomes

$$(51)\quad U-U_1\left(\frac{X}{N} - \frac{\partial \widetilde{Y}_1}{\partial N}\right) = W-W_1\left(\frac{Z}{M} - \frac{\partial \widetilde{Y}_2}{\partial M}\right) \ .$$

Since we know that free migration will lead to $U=W$ it is useful to rewrite (51) as

$$(52)\quad U_1\left(\frac{X}{N} - \frac{\partial \widetilde{Y}_1}{\partial N}\right) = W_1\left(\frac{Z}{M} - \frac{\partial \widetilde{Y}_2}{\partial M}\right) \ .$$

This expression differs from (34), the corresponding optimality condition when the objective function is stated in terms of average utility, in that the bracketed expressions in (52) are multiplied by the marginal private utility of private goods, and not by the marginal private utility of public goods (as in 34).

The next step is to compare (52) with the consequences of free migration where $U = W$. When all residents of a region consume the same quantity of private and public goods then equation (40) is the relevant benchwork for free migration. Equation (40) approximates the equation for the indifference curve passing through the points $(\frac{X}{N}, R_1)$ and $(\frac{Z}{M}, R_2)$:

$$(40) \quad (U_1+W_1)(\frac{X}{N} - \frac{Z}{M}) + (U_2+W_2)(R_1-R_2) = 0.$$

Equation (52) may be rewritten as follows:

$$(53) \quad U_1\frac{X}{N} - W_1\frac{Z}{M} + U_2R_1 - W_2R_2 = U_1\frac{\partial \widehat{Y}_1}{\partial M} - W_1\frac{\partial \widehat{Y}_2}{\partial M} + U_1\frac{R_1}{N} - W_1\frac{R_2}{M}$$

The terms $\frac{R_1}{N}$ and $\frac{R_2}{M}$ are the tax prices for the public good in the two regions. The left-hand side of (53) corresponds to the left-hand side of (40) in that the consumption quantities $(\frac{X}{N}, R_1)$ and $(\frac{Z}{M}, R_2)$ are weighted in (53) by the relevant marginal utilities and in (40) by the weighted average of the marginal utilities. The essential point is that the optimality condition in (53) contains (a) the differences in the marginal product of labour and (b) differences in tax prices, on the right-hand side of the equation. Inspection of (40) and (53) reveals that the crucial difference between these equations is given by the terms on the right-hand side. Free migration will not be guided by an "invisible hand" to achieve the most efficient distribution of population. From

(53) it may be seen that if free migration leads to

$U_1\frac{X}{N} - W_1\frac{Z}{M} + U_2R_1 - W_2R_2 = 0$ and if

$$U_1\left(\frac{\partial\widetilde{Y}_1}{\partial N} + \frac{R_1}{N}\right) - W_1\left(\frac{\partial\widetilde{Y}_2}{\partial M} + \frac{R_2}{M}\right) = 0$$

then optimality will obtain fortuitously. But if

$$U_1\left(\frac{\partial\widetilde{Y}_1}{\partial N} + \frac{R_1}{N}\right) - W_1\left(\frac{\partial\widetilde{Y}_2}{\partial M} + \frac{R_2}{M}\right) > 0$$

then population should move from region 2 to region 1, and vice versa.

Now let us evaluate what happens when there is a select migrant group in both regions but where the objective function continues to be total utility, as defined in (44). Each member of the migrant group is paid his marginal product less the tax price for public goods. Migratory equilibrium may be defined by

$$(54)\quad (U_1+W_1)\left(\frac{\partial\widetilde{Y}_1}{\partial N} - \frac{R_1}{N} - \frac{\partial\widetilde{Y}_2}{\partial M} + \frac{R_2}{M}\right) + (U_2+W_2)(R_1-R_2) = 0,$$

where the consumption quantities are evaluated by the weighted average marginal utilities. To facilitate comparison with (53) let the consumption quantities for the migrant group be weighted by the relevant marginal utilities in the two regions so that (54) may be rewritten as

$$(55)\quad U_2R_1 - W_2R_2 = U_1\left(\frac{R_1}{N} - \frac{\partial\widetilde{Y}_1}{\partial N}\right) - W_1\left(\frac{R_2}{M} - \frac{\partial\widetilde{Y}_2}{\partial M}\right).$$

or as

$$U_1\frac{\partial\widetilde{Y}_1}{\partial N} = W_1\frac{\partial\widetilde{Y}_2}{\partial M}.$$

Once again we see that free migration does not albcate population in an efficient manner, even when migrants are paid their marginal product.

Earlier in this section we examined a model in which the social welfare function for each region is defined in terms of the utility function of the representative person. This model is set forth, in terms of its objective function and its constraints, in equation (27). In our discussion of the model we gave equal weight to the social welfare functions of the two regions, by assuming $\lambda_1=1$. However, this procedure has the effect of giving a different weight to different individuals, depending upon where they happen to live. For certain purposes there is some logic in using such a weighting system. For example in many ways the member states or provinces of a federation are typically treated as equals in certain respects, by the central government, thus implying that the individual members of these provinces are given unequal treatment.[16] However, there does exist an intuitively appealing weighting system for the "representative-man" type of regional social welfare function. Each regional social welfare function may be weighted by the proportion of the national population in the region. The Lagrangian expression in (27) may be reformulated to read:

$$(56)\quad L_5 = \frac{N}{N_0}U\left(\frac{X}{N},R_1\right) + \frac{M}{N_0}W\left(\frac{Z}{M},R_2\right) + \theta_1[\widetilde{Y}_1(N)-X-R_1]$$
$$+ \theta_2[\widetilde{Y}_2(M)-Z-R_2] + \theta_3[N_0-N-M].$$

The solution for the maximization problem represented in (56) is precisely the same as the solution for the problem given in (44). When each "average utility" social welfare function is weighted by regional population, as a fraction of total population, the result is identical to that obtained by using total utility as the objective function. The remarks that were made concerning the total utility model apply here as well. If efficiency is defined in terms of the maximization of total utility, or total consumer surplus, then free migration will not in general produce an efficient solution. A central government which is committed to both free migration and efficiency will find it necessary to interfere with the signals given by the market.

It is tempting to view the decision-makers at the regional level as being mainly interested in maximizing the average utility of their region. Each region may be regarded as a club whose concern is to maximize the welfare of its members. In the process of reaching an efficient configuration of private and public good production the size of the membership group may change so that the optimizing policies of the club must accommodate the variability of population. Quite apart from the problem of non-optimal migration it is quite likely that, in a federation of regions, an inconsistency will exist between the objectives of the federal and regional governments. Since the federal government is concerned with the welfare of all members of the federation it will be primarily interested in maximizing total utility. If the federal government were in control of the supply of public and private goods at the regional level then the inter-regional configuration of output and population would be different. Each region will try to maximize average utility in the region while the federal government will be concerned with total welfare, over all regions.[17]

Our analysis of free migration has suggested that neither the federal government nor the regional governments will be satisfied with the consequences of the migration achieved by the free market. The federal government will be sensitive to the resultant non-optimality of the situation and may attempt to correct imbalances that exist in the inter-regional distribution of population. Without resorting to direct controls the primary instruments of federal policy are systems of subsidy and taxation, designed to alter incentives to migrate. For a region with relatively deficient population size the federal government can offer a conditional subsidy on private or public goods, and in so doing alter relative prices, or it can offer payments to individual residents which are unconditional upon the composition of regional output.

Let us derive a system of optimizing unconditional subsidies which may be used by the federal government to maximize total welfare. In so doing we shall adopt the condition for optimal migration given by (53) which will be restated as follows

$$(53)\quad U_1\frac{X}{N} - W_1\frac{Z}{M} + U_2R_1 - W_2R_2 = U_1\frac{\partial\widehat{Y}_1}{\partial N} - W_1\frac{\partial\widehat{Y}_2}{\partial M} + U_1\frac{R_1}{N} - W_1\frac{R_2}{M}\ .$$

Let (57) stand as an approximation to the free migration equilibrium condition in (40):

$$(57)\quad U_1\frac{X}{N} - W_1\frac{Z}{M} + U_2R_1 - W_2R_2 = \frac{U_1}{N}\widehat{Y}_1 - \frac{W_1}{M}\widehat{Y}_2 = 0.$$

Let $\frac{\widehat{V}_1}{N}$ and $\frac{\widehat{V}_2}{M}$ be the per capita federal subsidies in the two regions so that (57) becomes (58):

$$(58)\quad \frac{U_1}{N}(\widehat{Y}_1+\widehat{V}_1) - \frac{W_1}{M}(\widehat{Y}_2+\widehat{V}_2) = 0 \quad \text{or}$$

$$\frac{U_1}{N}\widehat{Y}_1 - \frac{W_1}{M}\widehat{Y}_2 = \frac{W_1}{M}\widehat{V}_2 - \frac{U_1}{N}\widehat{V}_1\ .$$

This enables us to write

$$(59)\quad \frac{W_1}{M}\widehat{V}_2 - \frac{U_1}{N}\widehat{V}_1 = U_1\frac{\partial\widehat{Y}_1}{\partial N} - W_1\frac{\partial\widehat{Y}_2}{\partial M} + U_1\frac{R_1}{N} - W_1\frac{R_2}{M} \quad \text{or}$$

$$\frac{\widehat{V}_1}{N} = \frac{W_1}{U_1}\left(\frac{\widehat{V}_2}{M} + \frac{\partial\widehat{Y}_2}{\partial M} + \frac{R_2}{M}\right) - \left(\frac{\partial\widehat{Y}_1}{\partial N} + \frac{R_1}{N}\right).$$

The point to be made here is that a system of federal subsidies can be devised to correct for misallocation of resources, provided that sufficient information is available.

This section has taken up the most simple model of multi-unit government and regional public goods and shown the need for intervention by a higher level of government. The model presented here has been a two-region model with identical individuals. Problems of non-benefit taxation have been ruled out by the use of a taxation system based on the marginal valuation or benefit from public goods. Inter-regional benefit and cost spillovers have also been ruled out by the assumption that all direct benefits and costs fall within the operative jurisdiction. In both regions it has been assumed that the regional public good is completely non-rivalrous within the region. In spite of these simplifying assumptions and the absence of Pigouvian externalities we have found that optimal allocation of population is not achieved without central intervention.

IV. Imperfect Migration in the Two-Region Model

In this section we relax the assumption that the nation or federation that is of interest to us contains a population of identical individuals. The analysis here remains within the framework of a two-region model but the residents of the two regions will have differentiated tastes. The existence of more than one fiscal region will enable product differentiation by the public sector of each region. As in the previous section each province or region produces a homogeneous private good and a single pure public good. Once again it is assumed that, within each region, each individual consumes an identical quantity of private and public goods. Also we continue to use a social welfare function for each region which is defined in terms of the average level of utility, or the welfare of the representative man. It is assumed that each region is concerned about the average level of utility in that region although the federal government will be concerned with the maximization of total utility over the whole federation.

The assumption of non-homogeneous individuals now allows us to consider how regional governments may attempt to alter the private-public goods product mix in order to influence population size. Each region will try to achieve an optimal population size and its product mix will serve as an instrument of policy. In the previous section each regional government responded passively to population flows. Since all individuals were assumed identical there was no advantage to be gained by altering the product mix from that dictated by the preferences of the resident population. In the present section the resident population will be subject to indirect control through the product mix. We shall specify this relationship in terms of a "demand for residence function" or "mobility function", as in equation (12) of section II. The mobility function for region I will be given by

$$(60)\quad N = \widetilde{N}\left(\frac{X}{N} - \frac{Z}{M},\ R_1, R_2\right)$$

and the corresponding function for region 2 will be

$$(61)\quad M = N_0 - \widetilde{N}\left(\frac{X}{N} - \frac{Z}{M},\ R_1, R_2\right)$$

where N_0 is the total fixed population of the two regions. The first argument of $\widetilde{N}$ is the difference between income net of tax in the two regions. The second argument is the level of public good consumption available to all residents in region I, while the third argument of $\widetilde{N}$ refers to the corresponding availability of public goods in province 2.[18]

As before, let it be assumed that the transformation curve is linear in both regions and let the unit of measurement for public goods be defined so that the unit price of public goods, in terms of private goods, is unity in both regions. The maximization problem confronting region I can be presented in the form of the following Lagrangian expression:

$$(62) \quad L_6 = U(\tfrac{X}{N},R_1) + \lambda_1[Y_1-X-R_1] + \lambda_2[\widehat{Y}_1(N)-Y_1]$$

$$+ \lambda_3[\widehat{N}(\tfrac{X}{N} - \tfrac{Z}{M},R_1,R_2)-N].$$

From this expression we can derive the first order conditions and these correspond to those that were derived in Section II. The condition for optimal population, corresponding to (18) is

$$(63) \quad (\frac{X}{N} - \frac{\partial \widehat{Y}_1}{\partial N})\frac{dN}{dR_1} = S_1N - 1 ,$$

where $S_1 = \frac{U_2}{U_1}$ and $\frac{dN}{dR_1}$ is the total derivative of N with respect to R_1. As was pointed out in Section II the product mix may be adjusted in such a way that the Samuelson condition is no longer satisfied. The analogous condition for region 2 is given by

$$(64) \quad (\frac{Z}{M} - \frac{\partial \widehat{Y}_2}{\partial M})\frac{dM}{dR_2} = S_2M-1 ,$$

where $S_2 = \frac{W_2}{W_1}$ and $\frac{dM}{dR_2}$ is the total derivative of M with respect to R_2.[19] The consequence of independent decision-making by the regional governments will be

$$(65) \quad (\frac{X}{N} - \frac{\partial \widehat{Y}_1}{\partial N})\frac{dN}{dR_1} - S_1N = (\frac{Z}{M} - \frac{\partial \widehat{Y}_2}{\partial M})\frac{dM}{dR_2} - S_2M.$$

The analysis up to this point is predicated upon the possibility of imposing indirect control over migration, through the regional product mix. The terms $\frac{dN}{dR_1}$

and $\frac{dM}{dR_2}$ are the parameters of the migration reaction function. As a result the action of the regional governments will lead to the satisfaction of equation (65). The federal government will be required by the public interest to ascertain the optimality of (65). The federal government may adopt one of two possible benchmarks of optimality and ask (a) is (65) consistent with the maximization of total utility or (b) is (65) consistent with the maximization of average utility in one region for a given level of average utility in the other? We shall take up the second question first. The question being posed asks whether independent maximization of average utility in each region will achieve efficiency in a more narrowly-defined sense than the maximization of total utility. Let the welfare maximization problem be set forth in the form of the following expression:

$$(66)\quad L_7 = U(\tfrac{X}{N},R_1) + \lambda_1[W(\tfrac{Z}{M},R_2)-W^0]$$

$$+ \lambda_2[\widetilde{Y}_1(N)-Y_1] + \lambda_3[\widetilde{Y}_2(M)-Y_2]$$

$$+ \lambda_4[Y_1-X-R_1] + \lambda_5[Y_2-Z-R_2]$$

$$+ \lambda_6[\widetilde{N}(\tfrac{X}{N} - \tfrac{Z}{M},R_1,R_2)-N] + \lambda_7[N_0-N-M].$$

From this Lagrangian the following two conditions can be derived:

$$(67)\quad S_1N-1 = [\tfrac{X}{N} - \tfrac{\partial\widetilde{Y}_1}{\partial N} - \lambda_1\tfrac{W_1}{U_1}\,\tfrac{N}{M}(\tfrac{Z}{M} - \tfrac{\partial\widetilde{Y}_2}{\partial M})]\tfrac{dN}{dR_1} \text{ and}$$

$$(68)\quad S_2 M - 1 = [(\frac{X}{N} - \frac{\partial \widetilde{Y}_1}{\partial N})\frac{U_1}{W_1}\,\frac{M}{N}\,\frac{1}{\lambda_1} - (\frac{Z}{M} - \frac{\partial \widetilde{Y}_2}{\partial M})]\frac{dN}{dR_2}$$

Independent optimization by each region will not generally lead to the satisfaction of (67) and (68). In our earlier analysis we found that for a single region the optimality condition for population may be written

$$(a)\quad \frac{X}{N} - \frac{\partial \widetilde{Y}_1}{\partial N} = 0 \quad \text{or}$$

$$(b)\quad (\frac{X}{N} - \frac{\partial \widetilde{Y}_1}{\partial N})\frac{dN}{dR_1} = S_1 N - 1 \quad ,$$

depending upon whether population is (a) directly or (b) indirectly controlled (through the product mix). For a two-region federation however optimality will require that, for each region, the two terms $(\frac{X}{N} - \frac{\partial \widetilde{Y}_1}{\partial N})$ and $(\frac{Z}{M} - \frac{\partial \widetilde{Y}_2}{\partial M})$ be considered in determining the public good output and population size.

Let us now consider the optimality conditions associated with the maximization of total utility in our two-region federation. In so doing it will be assumed that the federal government directs its policy instruments upon the problem of maximizing total utility. Let us form the relevant Lagrangian expression:

$$(69)\quad L_8 = NU(\tfrac{X}{N},R_1) + MW(\tfrac{Z}{M},R_2) + \pi_1[\widetilde{Y}_1(N)-Y_1] + \pi_2[\widetilde{Y}_2(M)-Y_2]$$

$$+ \pi_3[Y_1-X-R_1] + \pi_4[Y_2-Z-R_2] + \pi_5[\widetilde{N}(\tfrac{X}{N} - \tfrac{Z}{M},R_1,R_2)-N]$$

$$+ \pi_6[N_0-N-M].$$

This gives us the following first order conditions for the optimal product mix in the two regions:[20]

$$(70)\quad NS_1-1 = [\tfrac{X}{N} - \frac{\partial\widetilde{Y}_1}{\partial N} - (\tfrac{Z}{M} - \frac{\partial\widetilde{Y}_2}{\partial M})\frac{W_1}{U_1}]\frac{dN}{dR_1} \quad \text{and}$$

$$(71)\quad MS_2-1 = [\tfrac{X}{N} - \frac{\partial\widetilde{Y}_2}{\partial N} - (\tfrac{Z}{M} - \frac{\partial\widetilde{Y}_2}{\partial M})\frac{W_1}{U_1}]\frac{U_1}{W_1}\,\frac{dN}{dR_2} \quad .$$

If each of the two regions were maximizing average utility consistently with (63) and (64) then the federal government could use a system of optimizing subsidies to effect an efficient allocation of resources. For example the federal government might alter relative prices in order to change the product mix in each region. Let V_1 and V_2 be the subsidy per unit of public good, in regions 1 and 2 so that each region will choose a product mix for which the following conditions are satisfied:

$$(72)\quad S_1N+V_1-1 = \left(\frac{X}{N} - \frac{\partial\tilde{Y}_1}{\partial N}\right)\frac{dN}{dR_1}\text{ , and}$$

$$(73)\quad S_2M+V_2-1 = \left(\frac{Z}{M} - \frac{\partial\tilde{Y}_2}{\partial M}\right)\frac{dM}{dR_2}\text{ , where}$$

$$(74)\quad V_1 = \left(\frac{Z}{M} - \frac{\partial\tilde{Y}_2}{\partial M}\right)\frac{W_1}{U_1}\,\frac{dN}{dR_1}\quad\text{and}$$

$$(75)\quad V_2 = \left(\frac{X}{N} - \frac{\partial\tilde{Y}_1}{\partial N}\right)\frac{U_1}{W_1}\,\frac{dM}{dR_2}\quad .$$

These unit subsidies on the public good of each region are instruments of federal optimization policy which enable the federal authorities to re-allocate population between the two regions so as to maximize total welfare. The proper tool of federal policy here would be a system of conditional shared cost grants rather than unconditional grants. The model presented in this section depends upon the existence of a well-defined reaction function for population size, with respect to the product mix in each region. The subsidy system for federal optimization policy is contingent upon the specific behavioural assumptions made concerning the policy of regional governments. If regional governments produced an efficient output of public goods, in the Samuelson sense, so that $NS_1 = 1$ and $MS_2 = 1$, then the discussion of federal optimization policy in the previous section would be relevant for the present case.

V. Conclusion

The purpose of this paper has been to examine the efficiency of a world with regional public goods and population mobility. Distributional issues were suppressed by the assumption that each individual, in every region, consumes an identical quantity of private and public goods. The paper describes how a single region may determine its optimal population size by maximizing the utility level of its representative resident. In the analysis it was assumed that labour and land are the only factors of production. For a two-region federation with a fixed population it was shown that free migration would not lead to an optimal allocation of population between the two regions. This means that the Tiebout model does not have the efficiency properties that were originally claimed for it.

From the point of view of the federation the policy objective is to maximize social (federal) welfare. It was suggested above that regional governments are likely to be motivated to maximize the welfare of their average residents. Our analysis has shown that the federal authorities may be able to achieve a reconciliation of objectives through the use of federal subsidies. Section III dealt with a world of identical individuals and developed a system of unconditional subsidies to individuals to facilitate efficient migration flows. In Section IV we examined a world where individuals have different tastes and showed that the output of public goods might be affected by the responsiveness of migration flows to changes in the private-public good "product mix". If regional governments alter the product mix in order to take advantage of economies of scale in consumption of public goods by its residents then non-optimal migration flows will result. Our analysis has shown how a system of conditional federal subsidies may be designed to correct non-optimal migration.

This paper has been restricted by a number of assumptions that are made explicit throughout. It was assumed that the public good in each region is completely non-rivalrous, so that there is no congestion of public good benefits. Also, it has been assumed that there are no cost or benefit spillovers between regions. The analysis presented in this paper has been extended in two other papers to deal with congestion and benefit spillovers (Vardy, 1971a and 1971b). We have shown above that even when preferences are revealed and when both spillovers and congestion are ruled out there remains an important manifestation of "market failure" in the provision of regional public goods.

FOOTNOTES

1. See Samuelson (1954 and 1955).

2. See Buchanan (1950 and 1952) and Scott (1950 and 1952).

3. See Henry Aaron (1969).

4. Our interpretation of this and subsequent models owes much to an unpublished paper by Frank Flatters.

5. Inter-regional trade in private goods is ruled out throughout this paper. An important consequence of this is that no numeraire is provided to facilitate inter-regional comparison of values. This, of course, poses no problem in the present section, in which our concern centers on a single region.

6. This means that the transformation curve between private and public goods is linear. For constant returns to scale production functions this can be rationalized in terms of equal factor intensities for both goods. Note that we have defined the units of public goods so that their price, in terms of private goods, is unity.

7. Frank Flatters has shown that in a world of pure public goods population will be increased up to the point where the marginal product of labour is zero and that for a world of pure private goods population would be adjusted to the level at which the average and marginal products of labour are equal.

8. More precisely, the condition for optimal population would be

$$(10A)\quad U_1\left(\frac{X}{N} - \frac{\partial \widetilde{Y}}{\partial N}\right) = U.$$

9. Once again it is assumed that the second order conditions for a constrained maximum have been satisfied. Note that subscripts on $\widetilde{N}$ and U are used to denote partial derivatives.

10. Note that "social welfare" here is evaluated in terms of our specific welfare function, which is the utility function for the "representative" person.

11. For purposes of brevity we shall refrain from presenting a detailed discussion of the Lagrangian multipliers that appear in the remainder of the paper. The inter-

pretations which have been given in this section will provide a useful guide in understanding the shadow prices that turn up below.

12. Again, we assume that second-order conditions are satisfied.

13. It is because our model lacks a numeraire that comparisons of value between regions cannot be made without invoking cardinal utility and the possibility of inter-personal comparison. Note that this disreputable reliance on cardinality is implicit in the literature where benefits (or consumers' surplus) are measured. In order to add a dollar of benefit to consumer A to a dollar received by B we must assume that the marginal utility of money is the same for both individuals. B-G implicitly assume cardinality, as, for example, in their equation (1A).

14. The same indifference curve will apply to each individual by our assumption of identical individuals.

15. Equation (40) may be written

$$(40A) \quad \frac{(U_1+W_1)}{2}\left(\frac{X}{N} - \frac{Z}{M}\right) + \frac{(U_2+W_2)}{2}(R_1-R_2) = 0$$

to bring out the simple weighting system used to arrive at the average marginal utilities.

16. As one example of this consider the fact that each American state has two representatives in the United States Senate.

17. Note that the regional public goods discussed here are pure public goods within the region but benefits do not flow across regional boundaries.

18. The partial derivatives of $\widetilde{N}$ with respect to each of these arguments are given by $\widetilde{N}_1$, $\widetilde{N}_2$ and $\widetilde{N}_3$. By assumption $\widetilde{N}_1 > 0$, $\widetilde{N}_2 > 0$, and $\widetilde{N}_3 < 0$.

19. Note that $\frac{dM}{dR_2} = -\frac{dN}{dR_2}$ since $M = N_0 - N$.

20. In the derivation of (70) and (71) it is assumed that $U=W$, which is a necessary condition for equilibrium in choice of location.

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